A Fixed Point Theorem in Hausdorff Space

Rajesh Shrivastava, Manavi Kohli

Abstract: The aim of the present paper is to prove the existence of fixed point for any continuous mapping in Hausdorff spaces.

Keywords: Fixed point, Hausdorff space, mapping, continuous maping, contraction principle, minimax principle, commuting mapping

1. Introduction:

In 1972, Brouwer [1] proved his well known fixed point theorem and next Schauder extended the validity of Brouwer's fixed point theorem to normed linear spaces. Jungck [2],[3],[4] established some fixed and common fixed point theorems for continuous commuting mappings and gave criterion of the existence of fixed points for Cgf in compact metric spaces. Kakutani [5] generalized Brouwer's fixed point theorem to multimaps and applied the result to prove a version of the Von Neumann minimax principle in R_n. In this paper the results of Park[6] and, Singh and Rao [7] have also been extended.

2. Theorem:

Let T be a continuous mapping of a Hausdorff space

$$d(Tx,Ty) \le [\{\frac{\alpha d(x,Tx).d(y,Ty)+d(x,Ty).d(y,Tx)}{d(y,Ty)}\} + \{\beta\frac{d(y.Tx).d(y,Tx)+d(x,Ty).d(x,Tx)}{d(x,Ty)}\} + \{\gamma\frac{d(x,y).d(y,Tx)+d(x,y).d(x,Ty)}{d(y,Ty)+d(x,Tx)}\}] \quad ....(1)$$

X into itself and let $d:X\times X\to R^{+}$ be a continuous mapping such that for x, $y\in X$ and $x\neq y$, satisfying for all n=0,1,2,………..; $x,y\in X$ and $\alpha,\beta,\gamma\ge 0$. Also $0\le(\alpha+\beta+\gamma)<1$.

Then T has a unique fixed point.

- *Dr. Rajesh Shrivastava is Professor and Head of Department Mathematics in Govt. Science & Commerce College Benazir Bhopal (MP)*
  E-mail: rajeshraju0101@rediffmail.com
- *Mrs. Manavi Kohli is a research scholar in Department of Mathematics in Govt. Science & Commerce college Benazir, Bhopal (MP)*
  E-mail: manvi.kohli@rediffmail.com

Proof:

For any $x_0\in X$ we choose $x\in X$, we define a sequence $\{x_n\}$ of elements of X, such that

$x_{n+1}=Tx_n$, for n= 0,1,2,….

Now,

$$d(x_{n+1},x_{n+2})=d(Tx_n,Tx_{n+1})$$

From (1) we have

$$d(Tx_n,Tx_{n+1})\le[\{\alpha\frac{d(x_n,Tx_n).d(x_{n+1},Tx_{n+1})+d(x_n,Tx_{n+1}).d(x_{n+1},Tx_n)}{d(x_{n+1},Tx_{n+1})}\} + \{\beta\frac{d(x_{n+1},Tx_n).d(x_{n+1},Tx_n)+d(x_n,Tx_{n+1}).d(x_n,Tx_n)}{d(x_n,Tx_{n+1})}\} + \{\gamma\frac{d(x_n,x_{n+1}).d(x_{n+1},Tx_n)+d(x_n,x_{n+1}).d(x_n,Tx_{n+1})}{d(x_{n+1},Tx_{n+1})+d(x_n,Tx_n)}\}]$$

$$\le \{\alpha d(x_n,Tx_n)+\beta d(x_n,Tx_n)+\gamma d(x_n,Tx_n)\}$$
$$= d(x_n,Tx_n)\{\alpha+\beta+\gamma\}$$

Proceeding in the same manner, we get

$$d(x_n,x_{n+1})\le\{\alpha+\beta+\gamma\}^{n+1}d(x_0,x_1)$$

as $n\to\infty$, $\lim_{n\to\infty} d(x_n,x_{n+1})\to 0$

Hence,$\{x_n\}$ converges to limit x (say).By completeness of X, the sequence $\{x_n\}$ is Cauchy.

Claim:

x is a fixed point of T.

On the contrary if we assume that $x\neq Tx$, then

$$d(x,Tx)\le d(x,x_{n+1})+d(x_{n+1},Tx)$$
$$= d(x,x_{n+1})+d(Tx_n,Tx)$$

By using (1), as $n\to\infty$

$$d(x,Tx)\le 0$$

which is a contradiction.

Hence proved , x is the fixed point.

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3. Conclusion:

In this paper we have proved a fixed point theorem for contractive mapping. This work can further be used for establishing results for generalized contractive and contractive type set valued mapping in other metric spaces.

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